9
FORCING CHAINS AND GRID COLORING

On rare occasions in really hard puzzles you will still be stuck even if you try all of the previous methods. For those puzzles, you need to bring out the heavy artillery.

FORCING CHAINS

In a forcing chain, you examine all the possibilities of a situation and hope they all lead to one result. If they do, you know that result is true.

XY-WING

An XY-wing occurs when you have a situation like this:

XY-Wing

O	XY	O		XZ				
YZ			O	O	O			

We don't know whether cell 12 is an X or a Y, but we know it has to be one of those two. Let's try both possibilities, and see what happens to the cells with the circles in them.

XY-Wing With Cell 12 = X

O	X	O		Z				
YZ			O	O	O			

XY-Wing With Cell 12 = Y

O	Y	O		XZ				
Z			O	O	O			

In the case where cell 12 is X, cell 15 becomes a Z, which eliminates the Z candidates from the circled cells. In the case where cell 12 is a Y, cell 31 becomes a Z, and also eliminates the Z candidates from the circled cells. So no matter what the contents of cell 12, we can eliminate the Z candidates from the circled cells. That's an XY-wing.

Here's a puzzle that requires XY-wing:

Example 25

				2	7	8		
		3	5					2
5	2							4
							6	8
		8		4		7		
1	5							
6							3	1
2					5	4		
		7	3	9				

Using what we know up to this point, we arrive at this grid:

Example 25-1

49	1469	1469	169	2	7	8	5	3
78	78	3	5	16	4	169	19	2
5	2	169	1689	1368	13689	16	7	4
3479	479	249	129	5	139	123	6	8
39	69	8	1269	4	1369	7	12	5
1	5	26	7	368	368	23	4	9
6	89	5	4	7	28	29	3	1
2	3	19	168	168	5	4	89	7
48	148	7	3	9	128	5	28	6

At this point there are three different examples of XY-wings found.

1) Cells 72, 77, and 98 form an XY-wing. The candidates in these cells are 8 and 9, 2 and 9, and 2 and 8, respectively. Whether you choose the 2 or the 9 for cell 77, neither cell 91 nor 92 can be 8. If cell 77 is a 2, then cell 98 has to be an 8, which would eliminate 8's from the candidates for cells 91 and 92. If cell 77 is a 9 instead, then cell 72 has to be an 8, which again eliminates the 8's from the candidates for cells 91 and 92. No matter what cell 77 is, be it a 2 or a 9, we know that cells 91 and 92 cannot contain 8's. Once we remove those 8's, we know cell 91 is a 4.

2) Cells 76, 77, and 88 form an XY-wing. The candidates in these cells are 2 and 8, 2 and 9, and 8 and 9, respectively. Whether you choose the 2 or the 9 for cell 77, neither cell 84 nor 85 can be 8, so we can eliminate candidate 8 from cells 84 and 85. That leaves only one cell (88) in row 8 that can contain an 8.

3) Cells 11, 91, and 72 form an XY-wing. The candidates in these cells are 4 and 9, 4 and 8, and 8 and 9, respectively. Whether you choose the 4 or the 8 for cell 91, cell 12 can't be 9, so we can eliminate the 9 candidate from cell 12.

Doing the second XY-wing above is all you need to reach the solution using basic methods.

Example 25 Answer

9	6	4	1	2	7	8	5	3
8	7	3	5	6	4	1	9	2
5	2	1	8	3	9	6	7	4
7	4	2	9	5	1	3	6	8
3	9	8	2	4	6	7	1	5
1	5	6	7	8	3	2	4	9
6	8	5	4	7	2	9	3	1
2	3	9	6	1	5	4	8	7
4	1	7	3	9	8	5	2	6

XYZ-WING

To explain the XYZ-wing, it is easiest to use the term "buddy," so first I'll refresh your memory on what that is. A buddy is a cell that shares a row, column, or box with another cell. So the buddies of cell 43 are 13, 23, 33, 41, 42, 44, 45, 46, 47, 48, 49, 51, 52, 53, 61, 62, 63, 73, 83, and 93. Every cell has 20 buddies.

In an XYZ-wing, you're looking for a cell that has the candidates XYZ and that also has buddies that contain the candidates XZ and YZ. Any cell that's buddies to all three of the XYZ, XZ, and YZ cells cannot contain a Z.

Example 26

7								5
2	1	6		5			7	
				9			6	
1		8	2					
9		3				4		1
					4	3		9
	6			4				
	7			6		5	3	4
3								8

The XYZ-wing comes into play here:

Example 26-1

7	*38*	9	*136*	*12*	*26*	*18*	4	5
2	1	6	4	5	8	9	7	3
4	*38*	5	*13*	9	7	*18*	6	2
1	4	8	2	3	9	7	5	6
9	*25*	3	*567*	*78*	*56*	4	*28*	1
6	*25*	7	*15*	*18*	4	3	*28*	9
5	6	1	8	4	3	2	9	7
8	7	2	9	6	1	5	3	4
3	9	4	*57*	*27*	*25*	6	1	8

This is very similar to XY-wing except the three relevant cells contain candidates in the form XYZ, XZ, YZ rather than XY, XZ, YZ. Cells 54, 56, and 94 form an XYZ-wing. Whether cell 54 is a 5, 6, or 7, cell 64 can't be 5, so we can eliminate candidate 5 from cell 64.

Example 26 Answer

7	3	9	6	1	2	8	4	5
2	1	6	4	5	8	9	7	3
4	8	5	3	9	7	1	6	2
1	4	8	2	3	9	7	5	6
9	2	3	5	7	6	4	8	1
6	5	7	1	8	4	3	2	9
5	6	1	8	4	3	2	9	7
8	7	2	9	6	1	5	3	4
3	9	4	7	2	5	6	1	8

LONGER CHAINS

Sometimes you can find situations more complex than XY-wing or XYZ-wing where there are two possibilities, either of which leads to the same result. Here's an example:

Example 27

	5	1		6				9
				1				
			5			7	8	
		5					9	
	2		6	4	8		1	
	1					2		
	4	7			2			
				5				
9				3		8	4	

This leads to the following sticking point:

Example 27-1

8	5	1	7	6	*34*	*34*	2	9
2	7	*34*	8	1	9	6	5	*34*
34	9	6	5	2	*34*	7	8	1
6	*38*	5	2	7	1	*34*	9	*348*
37	2	9	6	4	8	5	1	*37*
47	1	*48*	3	9	5	2	6	*78*
5	4	7	9	8	2	1	3	6
1	*38*	*38*	4	5	6	9	7	2
9	6	2	1	3	7	8	4	5

At cell 51, if you choose the 3, it forces a 4 at cell 31, which forces a 3 at cell 23, which forces a 4 at 29. Back to cell 51. If instead you choose the 7, it forces a 3 at cell 59, which forces a 4 at 29. So in either case, cell 29 has to be 4. Knowing that gives you enough to finish off the puzzle.

Example 27 Answer

8	5	1	7	6	4	3	2	9
2	7	3	8	1	9	6	5	4
4	9	6	5	2	3	7	8	1
6	8	5	2	7	1	4	9	3
3	2	9	6	4	8	5	1	7
7	1	4	3	9	5	2	6	8
5	4	7	9	8	2	1	3	6
1	3	8	4	5	6	9	7	2
9	6	2	1	3	7	8	4	5

In other chains, you can reach a point where a cell has no possibilities.

Example 28

	7			6	8			
6					9	7		
			7				3	
5		2	4					
4			8		3			2
					5	9		8
	4				1			
		5	6					4
			3	2			9	

We get very close to the end before we stall.

Example 28-1

13	7	13	5	6	8	4	2	9
6	2	4	1	3	9	7	8	5
89	5	89	7	4	2	1	3	6
5	8	2	4	9	6	3	7	1
4	19	19	8	7	3	6	5	2
7	36	36	2	1	5	9	4	8
2	4	7	9	5	1	8	6	3
39	39	5	6	8	7	2	1	4
18	16	168	3	2	4	5	9	7

If you choose 1 at cell 13, it forces a 3 at cell 11. It also forces 9 at cell 53, which forces 8 at cell 33, which forces a 9 at 31. This is an impasse because now cell 81 has no possibilities. So we can exclude the 1 from cell 13, put in a 3 there, and zoom to the finish line.

Example 28 Answer

1	7	3	5	6	8	4	2	9
6	2	4	1	3	9	7	8	5
9	5	8	7	4	2	1	3	6
5	8	2	4	9	6	3	7	1
4	1	9	8	7	3	6	5	2
7	3	6	2	1	5	9	4	8
2	4	7	9	5	1	8	6	3
3	9	5	6	8	7	2	1	4
8	6	1	3	2	4	5	9	7

In still other chains, you can reach a point where there's no place for a certain number.

Example 29

	2		6			9		1
	6				9			7
					8		6	3
			8				3	
4								5
	5				2			
5	3		1					
7			2				1	
1		4			6		9	

This is how far you can get:

Example 29-1

38	2	5	6	34	7	9	48	1
38	6	1	34	2	9	48	5	7
9	4	7	5	1	8	2	6	3
2	17	9	8	45	15	47	3	6
4	17	38	9	6	13	78	2	5
6	5	38	347	347	2	1	48	9
5	3	2	1	9	4	6	7	8
7	9	6	2	8	35	35	1	4
1	8	4	37	357	6	35	9	2

If you choose 4 at cell 27, it forces an 8 at cell 18, which forces a 3 at cell 11, which forces a 4 at cell 15. This is an impasse because now there is no place for a 4 anywhere in row 4. So we can exclude the 4 from cell 27, making it an 8. From unfinishable to solved in one easy logical chain.

Example 29 Answer

8	2	5	6	3	7	9	4	1
3	6	1	4	2	9	8	5	7
9	4	7	5	1	8	2	6	3
2	7	9	8	5	1	4	3	6
4	1	8	9	6	3	7	2	5
6	5	3	7	4	2	1	8	9
5	3	2	1	9	4	6	7	8
7	9	6	2	8	5	3	1	4
1	8	4	3	7	6	5	9	2

GRID COLORING

In grid coloring, you pick a number and follow it around the grid to see what happens. It can be thought of as a "true-false test." Choose a candidate number from a cell and suppose that number is correct for that cell, and call that cell true. Then anywhere else that number appears within the cell's buddies, those cells are false. This in turn causes other appearances of that number to become true or false. If doing this leads to some contradiction or impossibility, then you can safely remove that candidate as a possibility from your starting cell. The reason that this technique is usually referred to as grid coloring is that this can be easily visualized using colors. For example, all the "true" cells might be colored blue, while the "false" cells are colored red. And in very advanced grid coloring techniques, a third color might even be used to indicate that the number might or might not appear in a certain cell (that is, it could be true *or* false).

TURBOT FISH

One basic grid coloring technique is known as turbot fish. It requires five cells that are arranged so that two pairs of cells are in the same rows, two pairs are in the same columns, and two cells are in the same box. If three or four of the pairs are the only places where a particular number can go, you have a turbot